



**PRELIMINARY CONSIDERATIONS IN THE DESIGN
OF A
LUDWIG TUBE FOR HIGH REYNOLDS NUMBERS**

L. J. Fila

ARO, Inc.

February 1968

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FOREWORD

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This technical report has been reviewed and is approved.

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ABSTRACT

Larger aircraft operate at Reynolds numbers above 10^7 which happens to be the upper limit of experimental information. Designers depend heavily on extrapolating available test results for minimum drag, maximum lift, and boundary-layer phenomena. Estimation of the last item is least reliable and involves the higher risks. There is a real need to extend experimental information to higher Reynolds numbers. Reynolds numbers of 10^8 are needed now; 4×10^8 can be projected as a need for 1980; and (10^9) is a possibility for 2010. If air is used as the wind tunnel fluid, the higher Reynolds numbers can be obtained almost exclusively by high pressures, static and dynamic. This causes the wind tunnel dimensions to be directly proportional to the Reynolds number. Practically, the wall thickness depends essentially on the Reynolds number and the diameter depends on model stresses. If a Ludwieg tube is contemplated, a minimum pressure of about 4 atm simultaneously establishes a maximum tube diameter and a minimum model stress. The length of a Ludwieg tube depends on specified time for a run and attaining uniform flow.

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SECTION I INTRODUCTION

Reynolds number is a dimensionless parameter which essentially represents the influence of viscosity. Both minimum drag and maximum lift are influenced by viscosity and both have a bearing on the design of an economical aircraft. The quantitative effect of Reynolds number on drag, lift, and certain stall characteristics has been established through the use of specially designed wind tunnels. After completion of such tests, these wind tunnels fall into disuse and obsolescence. Viscosity also exerts an effect by disturbing the flow which approaches the empennage and by interacting with a transonic shock wave to result in separation. Both of these phenomena involve stability and control, either of which may become unsatisfactory for safety or comfort. The designer needs to be fully informed on all these items in order to reduce the risks of investment.

During recent years a distinct Reynolds number gap has been allowed to develop. Low subsonic test data exist up to a Reynolds number* of 10^7 , and high subsonic data stop at 1.6×10^7 . The DC-3 airplane matches or exceeds both numbers in actual flight, but other larger commercial airplanes range up to Reynolds numbers of 6×10^7 . Still larger airplanes now in design will approach 8×10^7 , and shortly this may be expected to be raised by a future design. This establishes an urgent and immediate need for data on Reynolds number 10^8 phenomena.

Future needs can only be projected from past and current trends. The principal parameter, weight, of the airplane has doubled every ten years since 1940. If this persists, by 1983 the largest airplane may weigh 2,000,000 lb, and by 2003 a 10,000,000-lb airplane might be under consideration. Since the Reynolds number is directly proportional to wing chord and since the wing chord is nearly proportional to the square root of weight, flight Reynolds numbers are doubled every 20 years. Unfortunately, such a generalization on Reynolds number obscures significant departures. Flight Reynolds number depends on altitude as well as speed and size. Figure 1 illustrates a greater scope of flight Reynolds numbers. It relates gross weight and Reynolds number for Mach 0.9 flight at four altitudes (sea level, 30,000, 50,000, and 70,000 ft). Weights range from 10^4 to 10^7 lb, and Reynolds numbers range from 10^7 to 10^9 . The computations are based on the defining equation:

$$Re = \frac{\rho V c}{\mu}$$

*The Reynolds number magnitudes in this report are referred to the wing root chord.

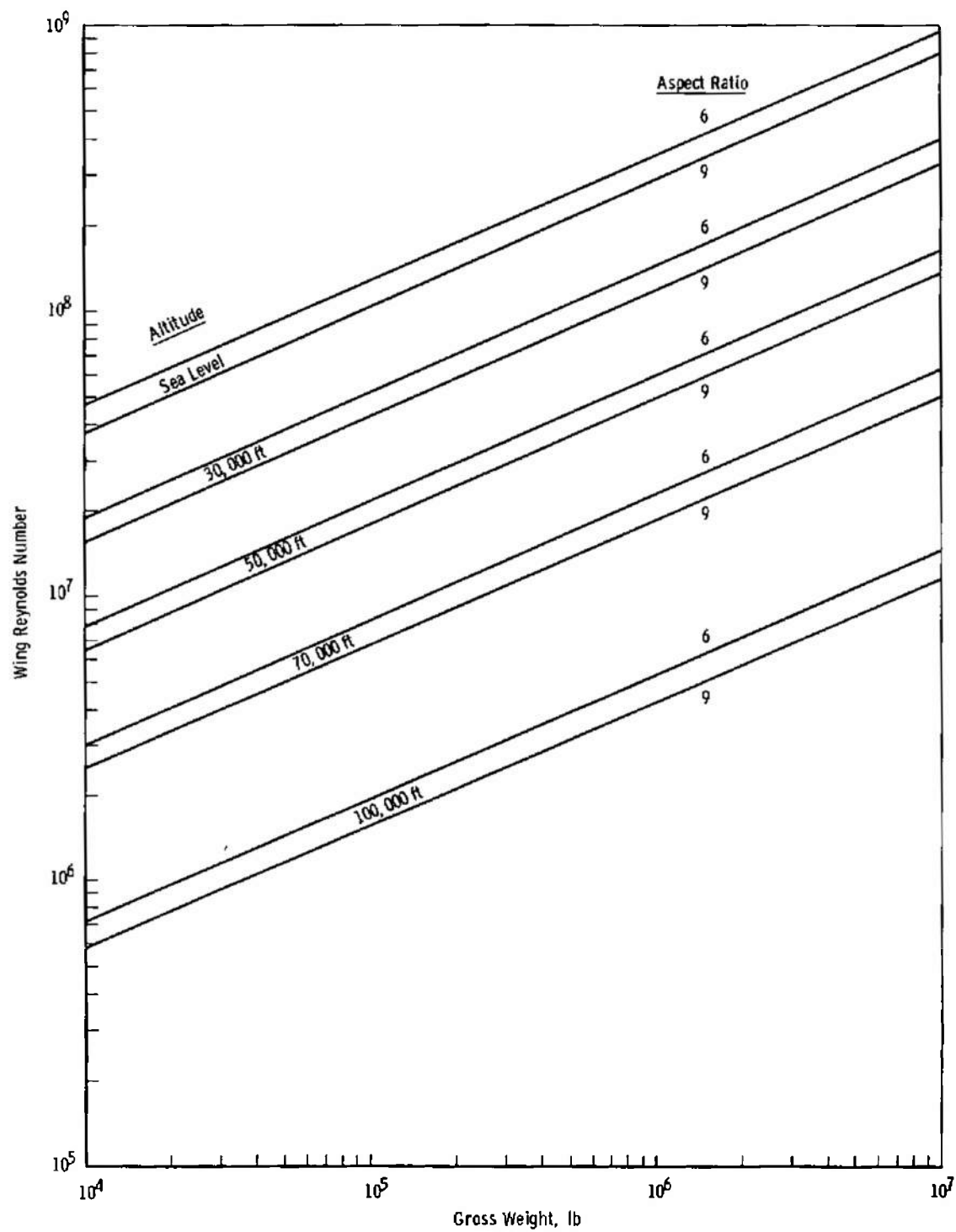


Fig. 1 Wing Reynolds Numbers for Mach 0.9 Flight

and the aspect ratio relationship for an elliptic planform,

$$c = \frac{4}{\pi} \left(\frac{W}{l_w AR} \right)$$

where

- Re is the Reynolds number
- ρ is the standard altitude density
- μ is the standard altitude viscosity
- V is the flight speed
- c is the root chord
- W is the gross weight
- l_w is the wing loading
- AR is the aspect ratio

Values of 6 and 9 were selected as practical extremes for aspect ratio. The wing loading was arbitrarily adjusted to gross weight according to the following table:

W	10^4	10^5	10^6	10^7
l_w	50	70	90	110

Other choices of numbers would produce different information, but the order of magnitude would remain the same. A modern airframe of 500,000 lb attains a Reynolds number of 6×10^7 near an altitude of 40,000 ft. It also is capable of attaining nearly the same speed at sea level, where the Reynolds number would exceed 2×10^8 . Admittedly, Mach 0.9 flights at sea level are unlikely, but nevertheless possible. Such considerations present difficulties in Reynolds number forecasts. Only an indefinite statement could be made. By about 1980, a two-million-pound airplane would probably operate at $Re = 10^8$ near 40,000 ft and possibly at $Re = 40 \times 10^8$ near sea level. Similarly, if present trends prevail to the year 2010, a ten-million-pound airplane would operate at $Re = 2 \times 10^8$ near 40,000 ft and possibly at $Re = 10^9$ near sea level.

Information from Fig. 1 suggests that short range needs should be based on a minimum Reynolds number of 10^8 and a maximum of 4×10^8 . Long range needs raise the upper limit to about 10^9 .

Concern has recently been expressed in regard to the influence of Reynolds Number on the aerodynamics of rocket airframes. This is a valid concern as control forces often become excessive and sensitive to Reynolds Number effects in the transonic regime. The Ludwig tube design considerations for rockets are qualitatively similar to those of airplanes, though they may result in a quantitatively different value.

SECTION II ANALYSIS

2.1 PRELIMINARY CONSIDERATIONS

Practical aspects of Reynolds number testing are based on the use of small inexpensive models in the controlled environment of a small wind tunnel. The loss in size (wing chord c) must be compensated by increases in density, ρ , and velocity, V , with decreases in viscosity, μ . When comparing different fluids, density depends on the number and mass of molecules, whereas viscosity depends more on the number of molecules in a given space. This would favor a choice of a heavy liquid. However, desired simulation of Mach 0.9 flow would require impractically high storage or driving pressures. Heavy gases such as the Freons® offer a slight gain (factor of three) over air; however, they also have the disadvantage of requiring high pressures. The saturation point of gases increases with density to the extent that high pressures are needed merely to avoid condensation during flow acceleration. The gain in Reynolds number is less than the increased pressure, for example, for Freons than for air. Thus, air still continues to be a practical ideal medium.

If air is selected, Reynolds gains can be achieved through increased pressure to raise the density. Control of temperature by refrigeration would also be helpful. This is clarified by the substitution of Ma for V . The symbol M is for Mach number and a is for the local acoustic speed. Both the acoustic speed and viscosity depend very nearly on the square root of temperature, thereby cancelling their temperature contribution to the Reynolds number. This leaves only the density to control the Reynolds number, which thus increases with a decrease in temperature. Control of Reynolds number by temperature alone is insufficient; high pressures are still necessary. As a consequence, practical considerations (cost and complexity of two controls) favor an initial choice of pressure control only. The remainder of this study is based on the choice of air and pressure control.

General consideration of wind tunnel size starts with the assumption that at least a half-wing model must be utilized. Boundary-layer separation is sensitive to twist, taper ratio, and sweepback as well as to thickness distribution and surface curvature. All are equally important so that two-dimensional modeling would have highly limited utility. Three-dimensional modeling is expedient if not necessary, and there is no practical alternative to a very large wind tunnel.

Size considerations must include large clearance space between model and wind tunnel walls. Interference effects between walls and a model cannot be eliminated, but they may be reduced to a small amount which

can be readily corrected by theoretical and experimental factors. Interference parameters of three different orders of magnitude must be considered in the present case. These are model span, thickness, and boundary layer. The influence of the latter may be easily hidden within the wall interference levels of the others. If a model dimension (e.g., thickness) is accepted as a first-order effect which can be corrected, then logically boundary-layer phenomenon becomes a secondary effect which nominally would require ten times the clearance that is needed for thickness. If a wind tunnel diameter-to-model span ratio of 1.5 is sufficiently large for data corrections because of model size, then a ratio of 15 would be proper to minimize interference of walls upon the model boundary layer. The large clearance, however, cannot be enforced because it would call for diameters in excess of 100 ft. This study is restricted to a half-span model and a diameter of 1.5 and 2.0 times the half span. Nevertheless, this imposes a limitation on the utility of such wind tunnels as well as introducing the risk that test results cannot be correlated with real flight phenomena. Primarily the failure of model testing would concern the critical conditions for insufficient separation.

A pressurized wind tunnel introduces another constraint on its size. The high pressure and density subject the model to higher aerodynamic loads. Matching of Reynolds number tends to preserve the total aerodynamic force, but geometric reduction increases the model stresses. In the case of a solid model the stresses increase inversely with the model volume. Consequently, the model size cannot be reduced below a certain limit. In this sense, model stresses determine the smallest wind tunnel diameter for the attainment of a specified Reynolds number.

Also, the high pressures which must be sustained by the tunnel walls establish an upper bound of Reynolds number which can be attained under prevailing design practices. The codes recommend 15,000 psi as a nominal design stress and 20,000 psi as a maximum stress. If these allowable values could be exceeded, higher Reynolds numbers could be realized.

The limits which are controlled by stresses will differ with the type of wind tunnel. This study is primarily concerned with a Ludwieg tube (Refs. 1 and 2) which has the advantages of an intermittent, blowdown wind tunnel. For subsonic flow, it has the advantage of a single chamber for storage at high pressure and for the test section as well. This advantage is diminished because the test section must sustain the high storage pressures. Also, a closed return tunnel should be capable of attaining a higher Reynolds number at lower pressures than could be developed in a Ludwieg tube. This is partly caused by the overpressure which must be sustained by the Ludwieg tube for the sonic nozzle flow into the atmosphere (or diffuser). The sonic nozzle reduces the flow rate in the test section for the same stagnation pressure.

The theory of the Ludwieg tube (Refs. 1 and 2) is fairly simple. For subsonic testing the tube consists of a long cylinder having one end closed by a diaphragm or gate. The exit area is reduced below the sectional area of the tube to provide a sonic throat. When the diaphragm is ruptured, Mach 1 conditions are established at the exit while a series of expansion waves (Ref. 3) proceed toward the closed end. The flow between the waves and the exit is subsonic and constant to satisfy the sonic flow conditions at the exit. The subsonic flow will remain the same until the reflected expansion waves from the fixed end return to the exit. If the initial storage pressure were sufficiently high, a new set of expansion waves would originate at the opening to produce another subsonic flow condition. The initial storage pressure in a Ludwieg tube is higher than in a conventional blowdown tunnel for the same rate of flow. In a Ludwieg tube the additional pressure is needed to move the expansion waves (or to keep all of the air from accelerating simultaneously).

Although this study is specifically directed to a Ludwieg tube, the conclusions apply qualitatively to all pressurized tunnels. In effect, this implies that there is some large Reynolds number for which scaled model testing is impractical if not impossible.

2.2 WIND TUNNEL WALL THICKNESS

The simple hoop stress on a cylindrical shell is:

$$2t\sigma = Dp_0$$

where

t = thickness of shell

σ = stress

D = diameter

and

p_0 = storage pressure

For a preliminary study, a constant modulus of elasticity is implied. Atmospheric pressure is neglected.

The tunnel dimensions are related to the Reynolds number, Re ,

$$Re = \frac{\rho V_c}{\mu}$$

The velocity is tentatively established to correspond to Mach 0.9. Since the acoustic speed depends on the static temperature, the velocity will depend upon the reservoir conditions in the tunnel. A reservoir temperature of 70°F is assumed. From this the static temperature of the flow is computed:

$$T = \frac{T_0}{\left(1 + \frac{\gamma-1}{2} M^2\right)} = 381^\circ R = -79^\circ F$$

where $T_o = 460 + 70 = 530^\circ\text{R}$
 $\gamma =$ ratio of specific heat of air
 $M = 0.9$

The acoustic speed is

$$a = 49.1 \sqrt{T} = 959 \text{ ft/sec}$$

so that the velocity becomes

$$V = Ma = 862 \text{ ft/sec}$$

Static temperature also establishes the value of viscosity

$$\mu = (338.5 + 0.575 T(^{\circ}\text{F})) 10^{-9} = 293 (10^{-9})$$

in pounds-seconds per square foot.

Density is expressed in terms of an unknown reservoir pressure, which is to be selected along with the wing chord to yield a desired Reynolds number. The static pressure is related to the reservoir pressure by

$$p = \frac{p_o}{\left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}} = \frac{p_o}{3.18}$$

and the density is established by the equation of state,

$$\rho = \frac{p}{gRT} = 6.93 (10^{-5}) p_o, \text{ slugs}$$

if p_o is expressed in pounds per square inch.

Now the Reynolds number is reduced to

$$Re = 2.04 (10^5) p_o c$$

in which c is expressed in feet.

Finally, upon substituting Reynolds number in the hoop stress equation, the thickness in inches becomes

$$\begin{aligned} t &= \frac{6 Re D}{2.04 (10^5) c} \\ &= \frac{6 Re}{2.04 (10^5)} \frac{D}{\left(\frac{b}{2}\right)} \left(\frac{b}{c}\right) \frac{1}{2} \end{aligned}$$

where b is the model wing span. The ratio of span to chord is defined by the aspect ratio, AR , for an elliptic wing:

$$\frac{b}{c} = \frac{\pi AR}{4}$$

which reduces the thickness to

$$t_{\sigma} = 1.155 (10^{-5}) \operatorname{Re} \left(\frac{2D}{b} \right) AR$$

Significantly, the thickness is independent of the diameter of the wind tunnel. It depends on the Reynolds number, the aspect ratio of the model, and the ratio of dimensions of the tunnel section to the model.

An appreciation of the influence of all quantities is gained from Figs. 2 and 3. Figure 2 is a plot of hoop stress versus Reynolds number. The lines are lines of constant wall thickness. Each thickness is represented by a pair of lines, one for a diameter-to-half-span ratio of 1.5 and another for a ratio of 2.0. Figure 2 applies for an aspect ratio of 9, and Fig. 3 is valid for an aspect ratio of 6.

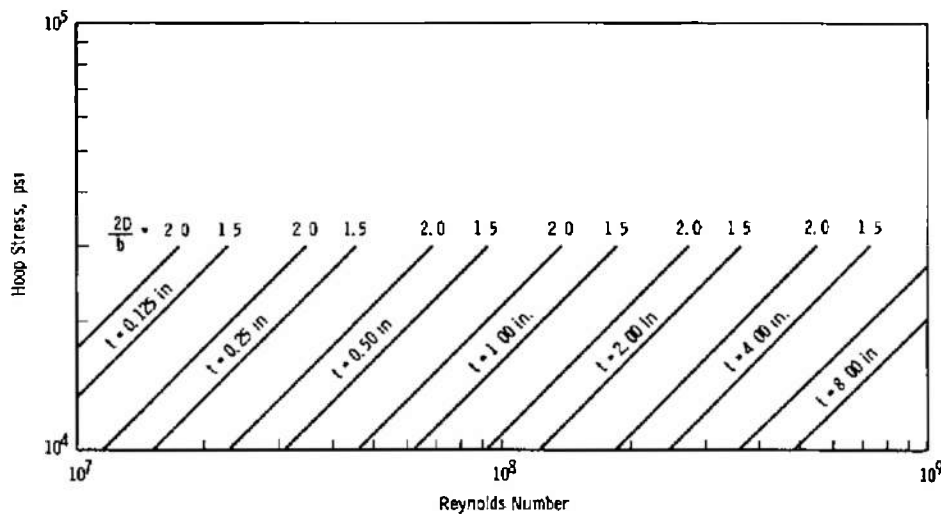


Fig. 2 Ludwig Tube Wall Thickness - Half-Span Model Aspect Ratio 9

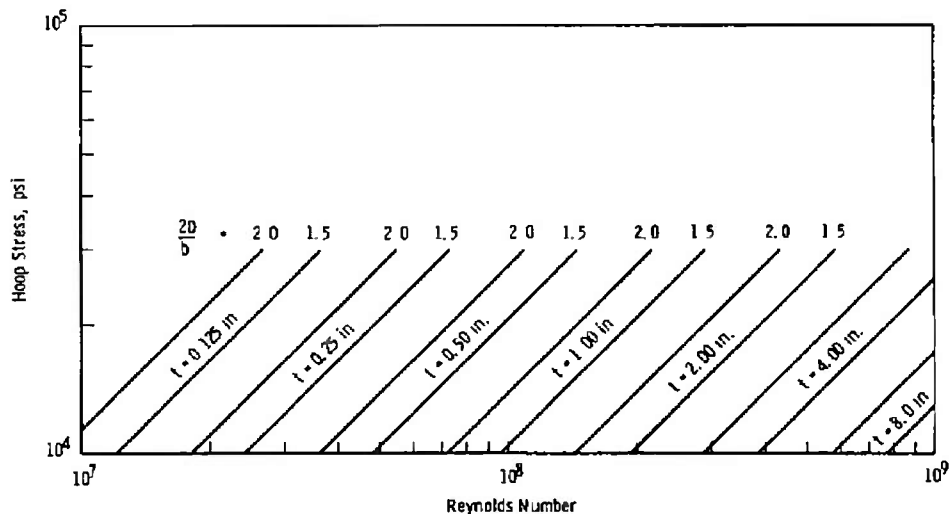


Fig. 3 Ludwig Tube Wall Thickness - Half-Span Model Aspect Ratio 6

From these two charts, the choices are restricted to a stress band between 10,000 and 20,000 psi according to accepted practices. For current purposes, a Reynolds number of 10^8 would be necessary; for the coming decade, 4×10^8 is predictable; and ultimately 10^9 may be a possible expectation. The charts indicate that wall thicknesses from 1 to 2 in. would be needed now, 3 to 5 in. would be needed in a few years, and thicknesses from 12 to 16 in. could be needed at some future time.

The charts were restricted to a maximum diameter-span ratio of 2.0 although larger ratios are highly desirable for boundary-layer studies. The smaller ratios were accepted as a compromise to reduce the thickness. This compromise, however, is not sufficient if a Reynolds number of 10^8 is to be exceeded. Higher design stresses would help. Although the thicknesses exceed what is regarded as ordinary, thickness could be removed as a limitation by recourse to other construction or type of tunnel.

2.3 WIND TUNNEL DIAMETER

Model stress is a real limitation or constraint on the diameter. The mean aerodynamic loading,

$$\frac{W}{S} = C_L \frac{\rho V^2}{2}$$

on the model is determined by the lift equation. However, wing loading on the model is independent of its weight (it is limited only by the design force of the supporting structure), but it is dependent on the angle of attack, auxiliary lift devices, and the moving fluid. Under the premise that auxiliary lift devices would not be tested at Mach 0.9, a maximum value, 1.6, is assumed for C_L . Arguments for a lower value may be pursued, but at a preliminary stage, the conservatism of a high value is preferred. Under these conditions, the loading on the model becomes

$$\frac{W}{S} = 41.2 p_\infty$$

The significance of this magnitude is best understood by recognizing a full-scale loading of five sea-level atmospheres for an actual airplane. The loading on the model is at least one order of magnitude greater, which produces the observation that reduction of model size while preserving Reynolds and Mach numbers fails to reduce surface loading.

For a cantilever support of a half-span wing, the location of maximum stress depends on wing geometry and distribution of structural elements for the model. If a solid model is assumed (the final computations justify the necessity of a solid structure) with a trapezoidal planform of 50-percent taper ratio, the computations are strongly simplified. Further simplification is realized by assuming a thickness ratio of 0.12 which,

although representative, may be somewhat high. With these assumptions, the maximum stress occurs at the support. Bending moments about the root are readily computed.

Stiffness at the root requires full knowledge of airfoil structural characteristics. An approximation is made by taking the average section modulus of an elliptic and a symmetrical, parabolic, lenticular section of the same dimensions. The section characteristic for an ellipse is

$$0.7854 \left(\frac{c}{2} \right)^3 \tau^2$$

and for the parabolic form

$$0.6095 \left(\frac{c}{2} \right)^3 \tau^2$$

The average of the two yields

$$0.0872 c^3 \tau^2 = 1.256 (10^{-3}) c^3$$

where τ is the thickness ratio. The moment, under an assumption of uniform loading, about the root section is

$$M = 41.2 p_0 \frac{b^2 c}{12}$$

With the aid of the simple flexure equation, the stress becomes

$$\sigma = \frac{41.2 p_0}{(12) (1.256) 10^{-3}} \frac{b^2}{c^2} \left(\frac{1}{144} \right)$$

in pounds per square inch. Aspect ratio for the "elliptic" wing is used again even though the stress analysis is based on a conical geometry. Admittedly, this involves an error of small magnitude which is admissible in a preliminary analysis. Also, the reservoir pressure is replaced by the Reynolds number:

$$\begin{aligned} p_0 \frac{b^2}{c^2} &= \frac{Re}{2.04 (10^5)} \frac{b^2}{c^3} \\ &= \frac{Re}{2.04 (10)^5} \left(\frac{b}{c} \right)^3 \left(\frac{2D}{b} \right) \frac{1}{D} \frac{1}{2} \end{aligned}$$

Thus the final form of the model flexural stress becomes

$$\sigma = 2.25 (10^{-5}) Re (AR)^3 \left(\frac{2D}{b} \right) \frac{1}{D}$$

This equation is reduced to a graph of Reynolds number versus tunnel diameter in Figs. 4 and 5. Figure 4 is based on an aspect ratio of 9, and Fig. 5 is restricted to an aspect ratio of 6. A straight line in each figure is for a constant stress and constant diameter-to-half-span ratio. Three stresses of 25,000, 50,000, and 100,000 psi and two diameter-span ratios of 1.5 and 2 are represented.

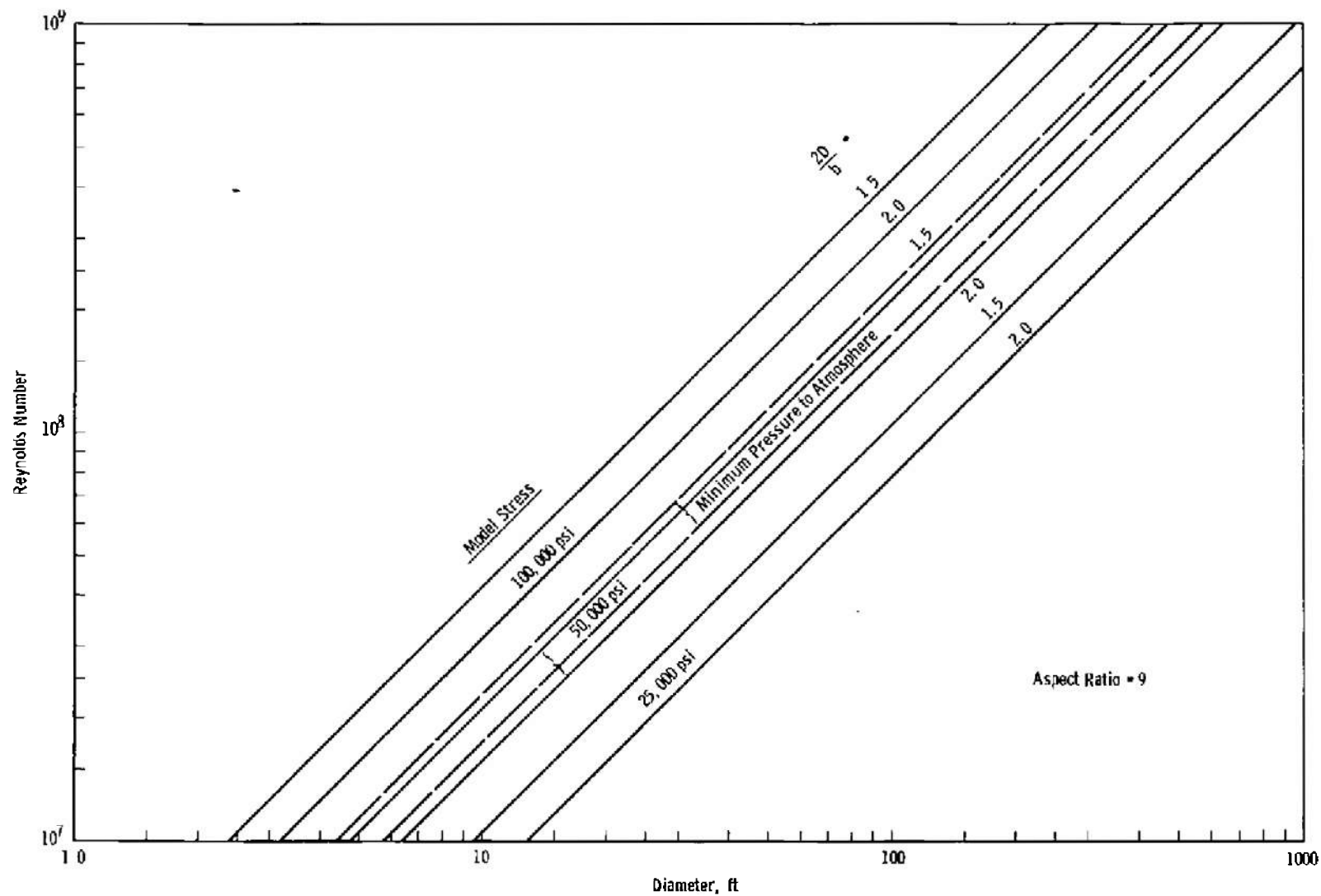


Fig. 4 Ludwig Tube Diameter - Half-Span Model Symmetrical Airfoil (Aspect Ratio 9)

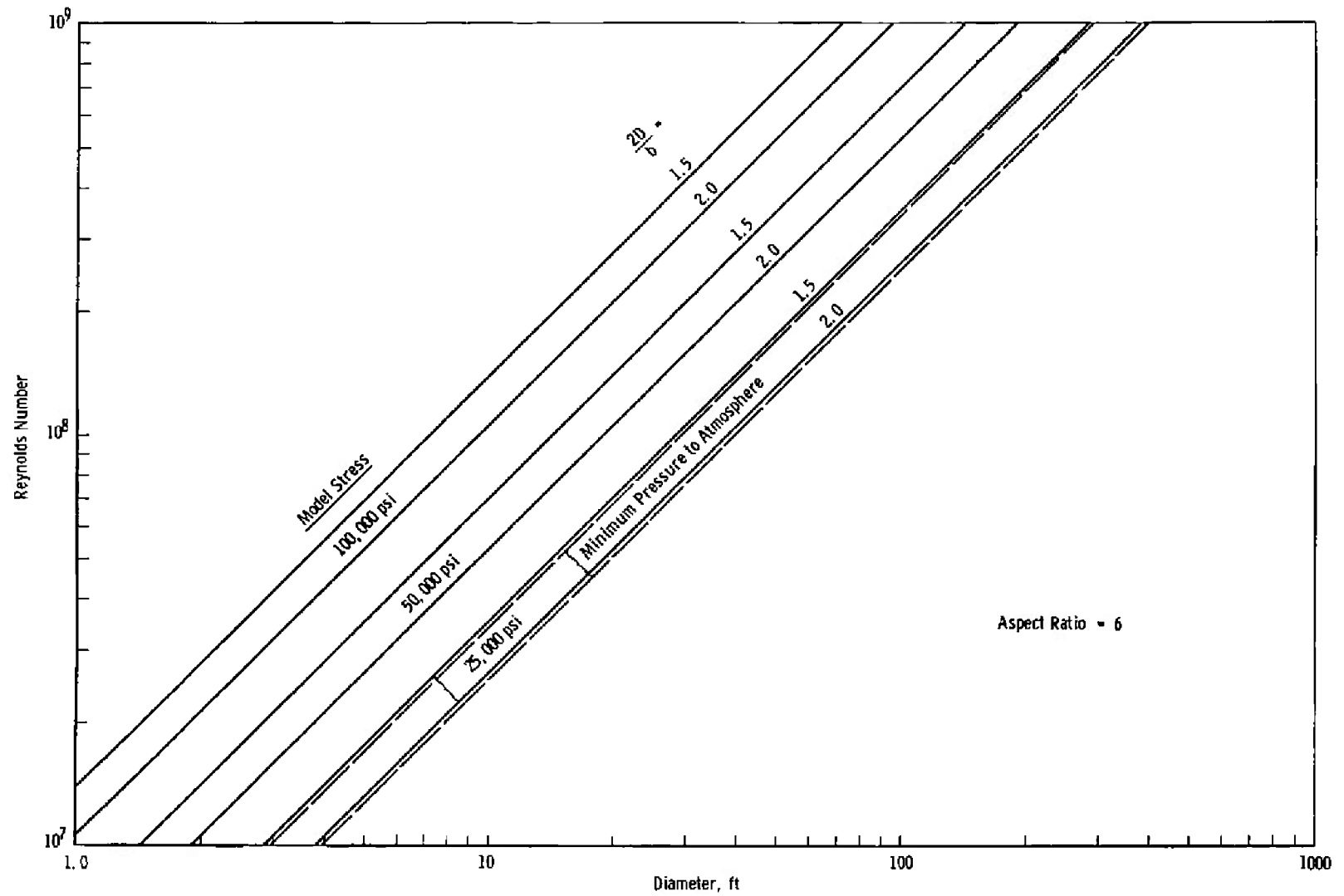


Fig. 5 Ludwig Tube Diameter - Half-Span Model Symmetrical Airfoil (Aspect Ratio 6)

The critical influence of model stress on the tunnel diameter is illustrated for a selected Reynolds number of 4×10^8 . A model stress of 25,000 psi would require a diameter of from 120 to 150 ft. This indicates several important considerations for a contemplated high Reynolds number test facility.

1. The tunnel design is seriously constrained by the model design.
2. Conventional concepts of model and tunnel design need to be compromised.

Some compromises are self-evident. Stress and diameter are inversely related. The increased difficulty of forming models with high strength steel and the necessity to recess instrumentation places an upper limit on admissible high stress material. Nevertheless, if the design stress can be raised from 25,000 to 50,000 psi, the wind tunnel diameter can be reduced by 50 percent. This would be a significant gain when starting with a 100-ft diameter.

Another gain is hidden in the choice of maximum lift coefficient. Transonic boundary-layer separation may be expected to occur at a lift coefficient of 0.8 or less. With a lift coefficient of 0.8 all diameters in Figs. 4 and 5 can be reduced by 50 percent. However, this choice of lift coefficients requires a precautionary red-line in testing. Measures must be introduced which will prevent or at least sound an alarm for any test configuration in excess of 0.8 lift.

Another compromise may be regarded as a sharp departure from conventional practices. This concerns the manner in which the model is supported. Instead of a cantilever suspension, two struts along the span would markedly reduce the flexural stress. Although this immediately raises objections because of flow interference, the influence on stress is worthy of investigation. One possibility is that of simple supports at locations where the stresses would be the same. This is easily achieved by recognizing that the moment is proportional to the third power of a dimension and that the section characteristic is also proportional to the third power. The planform is divided into two similar trapezoids, and the supports are placed at the centroid on each trapezoid. The moment at the dividing line is zero. (This could be reduced to a negative value, thereby reducing the stresses at the supports; but this is too involved to attempt for a preliminary study.) The dividing line is readily located at 0.5858 of the half-span from the root chord, and the supports are placed at the 0.2761 and 0.7811 locations of the centroids. The computation for the stress above each support is relatively simple but somewhat lengthy. Each of these stresses,

$$\sigma_2 = 0.17 \sigma$$

is approximately one-sixth that of the cantilever stress. This would reduce the tunnel diameter by a factor of six. The initially contemplated diameter of about 120 ft could be reduced to 20 ft by the struts alone. This gain is sufficiently great to encourage a second reflection on all objections to such compromise. All compromises together suggest the possibility of a 5-ft-diam tunnel that might be satisfactory.

Gains from additional supports become relatively smaller. A three-strut support serves as an example. The trapezoidal planform is divided into three similar sections by chords at 0.4131 and 0.7397 fraction of the semi-span. The three bays have centroids, which are arbitrarily selected for the strut locations, at 0.1985, 0.5702, and 0.8511 semi-spans. Stresses will be a maximum at or near the supports, and all three of these stresses will be equal. The stress,

$$\sigma_3 = 0.078 \sigma$$

will be approximately one-thirteenth that of the cantilever stress. Alone, the three strut supports would reduce a 120-ft diameter for a cantilever model to less than 10 ft, whereas the continuation of high strength material, limited maximum lift coefficient would make a 2.5-ft-diam tunnel a possibility.

Other diameter limitations arise from minimum pressure limitations, access to model, and structural limitations of the wind tunnel itself.

In order to maintain throat conditions at the exit, the least ideal storage pressure in the Ludwieg tube is defined by:

$$p_t = \frac{p_o}{3.18} \left(\frac{1 + 0.2 (0.9)^2}{1 + 0.2} \right)^{3.5} = \frac{p_o}{3.53}$$

With an atmospheric discharge pressure, $p_t = 14.7$ psia, this calls for a minimum storage pressure of 52 psia. Allowances for leakage and losses raise this value to 60 psia as an arbitrary choice.

For the exploration of this limitation, the Reynolds number equation is revised to the form

$$Re = 5.2 (10^5) \frac{p_o}{AR} \left(\frac{b}{2D} \right) D$$

which is added to Figs. 4 and 5 as dotted lines. These lines represent the largest possible diameter which may be selected to attain a specified Reynolds number. In Fig. 5, the maximum diameter lines are almost identical to the 25,000-psi stress lines, but on Fig. 4 the maximum diameter lines are close to the 50,000-psi model stress lines.

The significance of these near coincidences applies directly to the usefulness of a Ludwig tunnel. An example of an existing tunnel fixes the diameter and thickness, and the existence of a constructed model fixes the aspect ratio and span. For this situation the model stress reduces to a simple linear function

$$\sigma = K_1 Re = K_2 p_o$$

where K is a constant for the given wind tunnel and model combination. Now if the model happens to have an aspect ratio of 9 and the model stress is at its maximum allowable value corresponding to the minimum storage pressure, the model can be tested at one and only one value of Reynolds number. In order to obtain a range for testing, the design must satisfy the condition

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{Re_{max}}{Re_{min}} = \frac{p_{o_{max}}}{p_{o_{min}}} = \frac{\sigma_{max}}{K_2 p_{o_{min}}} \sim 10$$

Figure 4 becomes useful in this respect. For a given diameter, the minimum storage pressure fixes the minimum Reynolds number, and the maximum model stress fixes the largest Reynolds number. If Fig. 4 applies to a given wind tunnel and model, then a model stress of 50,000 psi restricts testing to one Reynolds number. A 100,000-psi stress allows testing over a ratio of

$$\frac{Re_{max}}{Re_{min}} = 2$$

and a 200,000-psi stress raises this ratio to 4. This again suggests that restricting the model load (limited lift coefficient during test or increased model support) is necessary in order to realize the nominal utility of the wind tunnel more fully.

Overlapping constraints would also influence the choice of model dimensions once the tunnel is built. There is almost no latitude in model dimensions because the minimum pressure would overload smaller models. In effect, the design of the wind tunnel and the design of models are directly interrelated. The minimum pressure influences the lower Reynolds number limit of operation while loading the model heavily. Stress limit of the model determines the upper Reynolds number for testing. Restricting the model load lowers the working stress at both limits proportionately. This strategem, when applied to the wind tunnel, increases the admissible diameter of the wind tunnel, but the tunnel and model designs remain interdependent.

Figure 4 indicates that a 5-ft-diam tunnel is the smallest which can be considered. This is based on minimum storage pressure for atmospheric discharge. All suggested compromises would call for larger diameters.

Under these observations, access to model ceases to be important for influence on dimensions.

2.4 WIND TUNNEL LENGTH

Length, the remaining dimension, depends on the time required for release of flow, time to attain steady flow, time to record measurements, and an appropriate ratio of length-to-diameter to produce uniform one-dimensional flow. This ratio has not been established because only small diameter Ludwig tubes have been developed with lengths that are more than ample for uniform flow. With a 5-ft-diam tube, previous length-to-diameter ratios cannot be justified because of cost. Now, it becomes expedient to undertake experimentation to establish a minimum acceptable length-to-diameter ratio. For the present an arbitrary ratio of 10 suggests lengths of 50 ft or more. Length and wave propagation velocity determine the time duration of constant flow conditions which persist until the wave travels the length of the tube twice. For a 50-ft length the duration is approximately

$$\frac{2(50)}{959} \sim 0.104 \text{ sec}$$

which is sufficient for reaching steady flow conditions and recording measurements after release. If half of this time is devoted to release, a mechanical method of opening the tube becomes a serious problem. This is illustrated by assuming an axial movement of a cover plate. Full flow begins when the plate is displaced through a distance of 0.25 diameter. If this is accomplished at constant acceleration for a 5-ft-diam tube in 0.05 sec, the acceleration attains a magnitude of 1000 ft/sec². This high value encourages the use of diaphragms which have been employed for small diameter openings. Some experimental work is needed to determine if either a single large diaphragm or a bank of smaller ones could operate satisfactorily.

SECTION III CONCLUSIONS AND OBSERVATIONS

For all practical purposes, design of a high Reynolds number test facility depends heavily on the design of models. Design limitations overlap to eliminate latitude in the design of either the facility or the model. Also, the limitations are sufficiently strong to render some testing impossible. Design latitudes and utility can be increased by compromising certain prevailing practices:

1. The maximum lift coefficient under test may be reduced to reduce the loads on the model. At Mach 0.9, a maximum lift coefficient of 0.8 should be adequate.
2. The model may be constructed from high strength steel.
3. Instead of a cantilever support, the model may have auxiliary supports (struts or cables) to reduce the model stresses.
4. The Ludwig tube may be designed to discharge into an evacuated chamber.

Some of these measures must be accepted as specifications for the tunnel design if a significant range of Reynolds number is to be tested. As an example, decrease of lift coefficient from 1.6 to 0.8 lowers the model stress by 50 percent which allows the use of a 100 percent larger diameter and doubles the test range. Higher allowable stresses raise the upper test Reynolds number proportionately. Auxiliary supports have the same influence as decrease of lift. Finally, a low pressure discharge lowers the minimum permissible storage pressure and increases the maximum permissible diameter. All of the compromises call for some decision before the tunnel design is undertaken.

Preliminary experimentation with length-to-diameter ratio and with the use of diaphragms is urgently needed.

A Ludwig tube for a high Reynolds number tunnel is feasible provided that adequate compromises to prevailing practices are made. Even with such compromises, some testing programs could not be undertaken either because of model or tunnel limitations.

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13. ABSTRACT Larger aircraft operate at Reynolds numbers above 10^7 which happens to be the upper limit of experimental information. Designers depend heavily on extrapolating available test results for minimum drag, maximum lift, and boundary-layer phenomena. Estimation of the last item is least reliable and involves the higher risks. There is a real need to extend experimental information to higher Reynolds numbers. Reynolds numbers of 10^8 are needed now; 4×10^8 can be projected as a need for 1980; and (10^9) is a possibility for 2010. If air is used as the wind tunnel fluid, the higher Reynolds numbers can be obtained almost exclusively by high pressures, static and dynamic. This causes the wind tunnel dimensions to be directly proportional to the Reynolds number. Practically, the wall thickness depends essentially on the Reynolds number and the diameter depends on model stresses. If a Ludwig tube is contemplated, a minimum pressure of about 4 atm simultaneously establishes a maximum tube diameter and a minimum model stress. The length of a Ludwig tube depends on specified time for a run and attaining uniform flow.		

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